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Natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption

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Abstract

The effect of internal heat generation/absorption on a steady two-dimensional natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. The equations are mapped into the domain of flat vertical plate, and then solved numerically employing the implicit finite difference method, known as Keller-box scheme. Effects of the pertinent parameters, such as the heat generation/absorption parameter (Q), the amplitude of the waviness (α) of the surface and Prandtl number Pr on the rate of heat transfer in terms of the local Nusselt number (Nu_x), isotherms and the streamlines are discussed.

Keywords: Natural convection; Uniform surface temperature; Heat generation or absorption; Keller box; Heat transfer; Vertical wavy surface

1. Introduction

The natural convection about a heated vertical wavy surface has received a great deal of attention due to its relation to practical applications of complex geometries. It is also a model problem for the investigation of heat transfer from roughened surfaces in order to understand heat transfer enhancement. Yao [1] studied the case of uniform surface temperature laminar free convection along a semi-infinite vertical wavy surface. The sinusoidal wavy surface can be viewed as an approximation too much practical geometries in heat transfer. A good example is a cooling fin. Since cooling fins have a larger area than a flat surface, they are better heat transfer devices. Another example is a machine-roughened surface for heat transfer enhancement. The interface between concurrent or countercurrent two-phase flow is another example remotely related to this problem. Such an interface is always wavy and momentum transfer across it is by no means similar to that across a smooth, flat surface, and neither is the heat transfer. Also a wavy interface can have an important effect on the condensation process.

Yao [1] and Moulic and Yao [2,3] studied the effect of surface waviness on the natural convection boundary layer. Hossain and Pop [4] investigated the magneto-hydrodynamic boundary layer flow and heat transfer along a continuous moving wavy surface. Alam et al. [5] have also studied the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field. On the other hand, Hossain and Rees [6] have investigated the combined effect of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface. The effect of waviness of the surface on the heat and mass flux is investigated in combination with the species concentration for a fluid having Prandtl number equal to 0.7. Munir et al. [7,8] investigated natural convection with variable viscosity and thermal conductivity along a vertical wavy cone. Recently, Kabir et al. [9] have studied the problem of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. In fact, the literature is replete with examples dealing with the heat

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Nomenclature

C_p	specific heat at constant pressure	$J \cdot kg^{-1} \cdot K^{-1}$
f	dimensionless stream function	
g	acceleration due to gravity	$m \cdot s^{-1}$
Gr	Grashof number	
h	heat flux coefficient	
k	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
L	characteristic length associated with the wavy surface	m
Nu_x	local Nusselt number	
P	pressure of the fluid	$N \cdot m^{-2}$
Pr	Prandtl number	
Q_0	constant	
Q	heat generation/absorption parameter	
q_w	heat flux at the surface	$W \cdot m^{-2}$
T	temperature of the fluid in the boundary layer	K
T_∞	temperature of the ambient fluid	K
T_w	temperature at the surface	K
u, v	the dimensionless x- and y-component of the velocity	$m \cdot s^{-1}$
$\hat{u}, \hat{v}$	the dimensional $\hat{x}$- and $\hat{y}$-component of the velocity	$m \cdot s^{-1}$
x, y	axis in the direction along and normal to the tangent of the surface	

Greek symbols

α	amplitude of the surface waves	
β	volumetric coefficient of thermal expansion	K^{-1}
ψ	stream function	$m^2 \cdot s^{-1}$
η	non-dimensional similarity variable	
ρ	density of the ambient fluid	$kg \cdot m^{-3}$
ν	kinematic coefficient of viscosity	$m^2 \cdot s^{-1}$
μ	dynamic coefficient of viscosity	$kg \cdot m^{-1} \cdot s^{-1}$
θ	dimensionless temperature function	
$\sigma(x)$	surface profile function defined in (1)	

Subscripts

w	wall conditions
∞	ambient temperature
x	differentiation with respect to x

Superscript

$'$	differentiation with respect to η

transfer in laminar flow of viscous fluids. Vajravelu and Hadjinolaou [10], studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. In this study, Vajravelu and Hadjinolaou [10] considered that the volumetric rate of heat generation, q''' [$W \cdot m^{-3}$], should be

$$q''' = \begin{cases} Q_0(T - T_\infty), & \text{for } T \geqslant T_\infty \\ 0, & \text{for } T < T_\infty \end{cases}$$

where Q_0 is the heat generation/absorption constant. The above relation explained by Vajravelu and Hadjinolaou [10], is valid as an approximation of the state of some exothermic process and having T_∞ as the onset temperature. When the inlet temperature are not less than T_∞ they used $q''' = Q_0(T - T_\infty)$.

In this paper, attention has been given to a study of the natural convection flow of a viscous incompressible fluid along a heated vertical wavy surface with a distributed heat source as given in [10] for $T > T_\infty$. Here the surface temperature T_w is higher than the ambient temperature T_∞. Using the appropriate transformations, the boundary layer equations are reduced to non-linear partial differential forms. The transformed boundary layer equations are solved numerically using the implicit finite difference method known as Keller box elimination technique [11]. The effect of varying the heat generation/absorption on the heat transfer rate in terms of local Nusselt number as well as on the streamlines and isotherm patterns is shown graphically.

2. Formulation of the problem

The boundary layer analysis outlined below allows $\hat{\sigma}(\hat{x})$ being arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$\hat{y}_w = \hat{\sigma}(x) = \alpha \sin\left(\frac{n\pi \hat{x}}{L}\right) \tag{1}$$

where L is the characteristic length associated with the wavy surface.

The geometry of the wavy surface and the two-dimensional Cartesian coordinate system are shown in Fig. 1.

Under the usual Boussinesq approximation, we consider the flow governed by the following equations:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \tag{2}$$

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{x}} + \nu\nabla^2\hat{u} + g\beta(T - T_\infty) \tag{3}$$

$$\hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} = -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{y}} + \nu\nabla^2\hat{v} \tag{4}$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} = \frac{k}{\rho C_p}\nabla^2 T + \frac{Q_0}{\rho C_p}(T - T_\infty) \tag{5}$$

where $(\hat{x}, \hat{y})$ are the dimensional coordinates along and normal to the tangent of the surface and $(\hat{u}, \hat{v})$ are the velocity components parallel to $(\hat{x}, \hat{y})$, $\nabla^2(= \partial^2/\partial x^2 + \partial^2/\partial y^2)$ is the Laplacian, g is the acceleration due to gravity, $\hat{p}$ is the dimensional pressure of the fluid, ρ is the density,

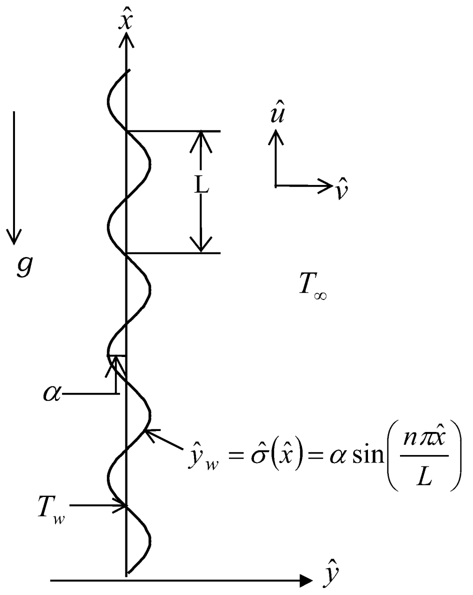

Fig. 1. Physical model and coordinate system.

k is the thermal conductivity, C_p is the specific heat at constant pressure and $\nu(=\mu/\rho)$ is the kinematic viscosity and μ is the dynamic viscosity of the fluid in the boundary layer region. The amount of heat generated or absorbed per unit volume is $Q_0(T-T_\infty)$, Q_0 being a constant, which may take either positive or negative values. The source term represents the heat generation when $Q_0 > 0$ and the heat absorption when $Q_0 < 0$.

The boundary conditions for the present problem are

$$\begin{aligned} &\hat{u}=0, \quad \hat{v}=0, \quad T=T_w \quad \text{at } \hat{y}=\hat{y}_w=\sigma(\hat{x}) \\ &\hat{u}=0, \quad T=T_\infty \quad \hat{p}=p_\infty \quad \text{as } \hat{y}\to\infty \end{aligned} \tag{6}$$

where T_w is the surface temperature, T_∞ is the ambient temperature of the fluid.

Following Yao [1], we now introduce the following non-dimensional variables:

$$\begin{aligned} &x=\frac{\hat{x}}{L}, \qquad y=\frac{\hat{y}-\hat{\sigma}}{L}Gr^{1/4} \\ &u=\frac{\rho L}{\mu}Gr^{-1/2}\hat{u}, \qquad p=\frac{L^2}{\rho\nu^2}Gr^{-1}\hat{p} \\ &v=\frac{\rho L}{\mu}Gr^{-1/4}(\hat{v}-\sigma_x\hat{u}), \qquad \theta=\frac{T-T_\infty}{T_w-T_\infty} \\ &\sigma_x=\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{x}}=\frac{\mathrm{d}\sigma}{\mathrm{d}x}, \qquad Gr=\frac{g\beta(T_w-T_\infty)}{\nu^2}L^3 \end{aligned} \tag{7}$$

where θ is the dimensionless temperature function. The (x, y) are not orthogonal, but a regular rectangular computational grid can be easily fitted in the transformed coordinates. It is also worthwhile to point out that (u, v) are the velocity components parallel to (x, y) which are not parallel to the wavy surface.

Introducing the above dimensionless dependent and independent variables into Eqs. (2)–(5) following dimensionless form of the governing equations are obtained after ignoring terms of smaller orders of magnitude in Gr, the Grashof number defined in (7).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial x}+\sigma_x Gr^{1/4}\frac{\partial p}{\partial y}+\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}+\theta \tag{9}$$

$$\begin{aligned} &\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\sigma_{xx}u^2 \\ &\quad=-Gr^{1/4}\frac{\partial p}{\partial y}+\sigma_x\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2} \end{aligned} \tag{10}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\left(1+\sigma_x^2\right)\frac{\partial^2\theta}{\partial y^2}+Q\theta \tag{11}$$

In the above equations, Pr and Q are, respectively known as the Prandtl number and the heat generation/absorption parameter, which are defined as

$$Pr=\frac{\nu}{\alpha}, \qquad Q=\frac{Q_0L^2}{\mu C_p}Gr^{-1/2} \tag{12}$$

It can easily be seen that the convection induced by the wavy surface is described by Eqs. (8)–(11). We further notice that, Eq. (10) indicates that the pressure gradient along the y-direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along x-direction can be determined from the inviscid flow solution. For the present problem this pressure gradient is zero. Eq. (10) further shows that $Gr^{-1/4}\partial p/\partial y$ is O(1) and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from Eqs. (9) and (10) leads to

$$\begin{aligned} &u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} \\ &\quad=\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2+\frac{1}{1+\sigma_x^2}\theta \end{aligned} \tag{13}$$

The corresponding boundary conditions for the present problem then turn into

$$\begin{aligned} &u=v=0, \quad \theta=1 \quad \text{at } y=0 \\ &u=0, \quad \theta=0, \quad p=0 \quad \text{as } y\to\infty \end{aligned} \tag{14}$$

Now we introduce the following transformations to reduce the governing equation to a convenient form:

$$\psi=x^{3/4}f(x,\eta), \qquad \eta=x^{-1/4} \qquad \theta=\theta(x,\eta) \tag{15}$$

where η is the pseudo similarity variable and ψ is the stream-function that satisfies Eq. (8) and is defined by

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x} \tag{16}$$

Introducing the transformations given in Eq. (15) into Eqs. (13) and (11) we have

$$\begin{aligned} &\left(1+\sigma_x^2\right)f'''+\frac{3}{4}ff''-\left(\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2+\frac{1}{1+\sigma_x^2}\theta \\ &\quad=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \end{aligned} \tag{17}$$

$$\frac{1}{Pr}(1+\sigma_x^2)\theta'' + \frac{3}{4}f\theta' + x^{1/2}Q\theta = x\left(f'\frac{\partial\theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \tag{18}$$

The boundary conditions (14) now take the following form:

$$\begin{aligned} &f(x,0) = f'(x,0) = 0, \qquad \theta(x,0) = 1 \\ &f'(x,\infty) = \theta(x,\infty) = 0 \end{aligned} \tag{19}$$

Solutions of local non-similar partial differential equations (17), (18), subject to the boundary conditions (19), are obtained by using implicit finite difference method developed by Keller [11]. This method has extensively been used recently by Hossain et al. [4–9] and hence the details of this method have not been discussed here.

However, once we know the values of the function f and θ and their derivatives, it is important to calculate the values of the local Nusselt number, Nu_x from the following relation:

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} \tag{20}$$

where

$$q_w = -k(\hat{n}\cdot\nabla T)_{y=0} \tag{21}$$

Using the transformation (15) Nu_x takes the following form

$$Nu_x(Gr/x)^{-1/4} = -\sqrt{1+\sigma_x^2}\,\theta'(x,0) \tag{22}$$

Finally, it should be mentioned that for the computational purpose the period of oscillations in the waviness of this surface has been considered to be π. But for comparison purpose with Yao [1], typical values of n have been taken to be 2.

3. Results and discussion

In this paper, The effect of internal heat generation/absorption on a steady two-dimensional natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated using the very efficient implicit finite difference method known as Keller box scheme [11]. Here we discuss the numerical results obtained from Eqs. (17)–(19) using the method mentioned above. It can be seen that the solutions are affected by three parameters, namely the heat generation/absorption parameter Q, Prandtl number Pr and the amplitude of the wavy surface α. So we focus our attention on the effect of Q, Pr and α on the rate of heat transfer in terms of the local Nusselt number Nu_x measured from the relation (22).

Since values of $\theta'(x,0)$ are known from the solutions of the coupled equations (17) and (18), numerical values of the local heat transfer, $Nu_x(Gr/x)^{-1/4}$ from (22) are calculated for a wide range of the axial distance variable x starting from the leading edge. Numerical values of $Nu_x(Gr/x)^{-1/4}$ thus obtained for different values of Q, Pr and α are depicted in Figs. 2–5, respectively.

At first it should be noted that, in absence of the heat generation/absorption parameter in the flow field (i.e., $Q = 0.0$), we recover the problem discussed by Yao [1] considering the form $\sigma(x) = \alpha \sin(2\pi x)$ for $Pr = 1.0$.

The effect of Prandtl number Pr, on the rate of heat transfer $\theta'(x,0)$ is shown in Fig. 2 and Fig. 3 for $Q = 0.0$ and $Q > 0$, respectively, while $\alpha = 0.3$. The reduced rate of heat transfer $\theta'(x,0)$ varies according to the slope of the wavy surface. This is due to the alignment of the buoyancy force $1/(1+\sigma_x^2)$, as shown in Eq. (17), which drives the flow tangentially to the wavy surface. Fig. 2 shows that without heat generation/absorption the rate of heat transfer $\theta'(x,0)$ and their amplitude reduce at a great extent for decreasing values of Pr. From Fig. 3 we observe that for the influence of heat generation, the decreasing rate of heat transfer becomes slower in the downstream region when Pr is small.

The effect of internal heat generation/absorption on the rate of heat transfer from the wavy surface while $\alpha = 0.3$ and $Pr = 0.01$ (liquid metal) is illustrated in Fig. 4. We see that the rate of heat transfer from the heated surface decreases with the increase of the heat generation parameter. This is

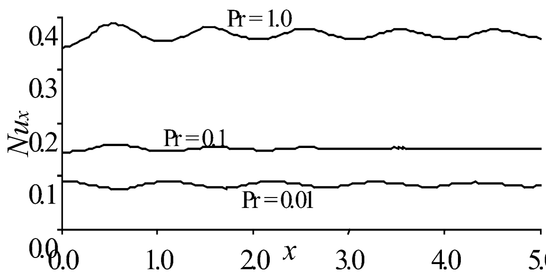

Fig. 2. Rate of heat transfer for different values of Pr while $Q = 0.0$ and $\alpha = 0.3$.

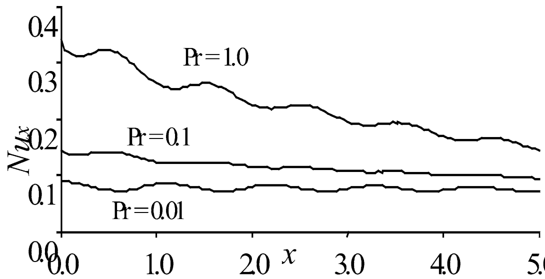

Fig. 3. Rate of heat transfer for different values of Pr while $Q > 0$ ($Q = 0.1$) and $\alpha = 0.3$.

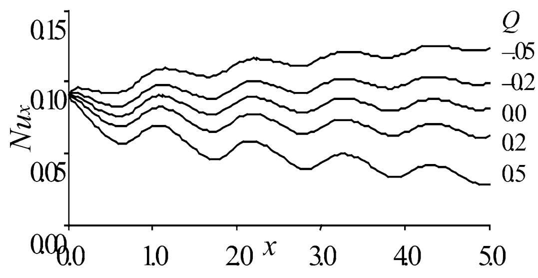

Fig. 4. Rate of heat transfer for different values of Q while $Pr = 0.01$ and $\alpha = 0.3$.

expected, since the heat generation mechanism will increase the fluid temperature near the surface. On the other hand, the presence of heat absorption ($Q < 0$) creates a layer of cold fluid adjacent to the heated surface and therefore the heat transfer rate from the surface increases. The amplitude of the rate of heat transfer for heat generation case is slightly higher than that of the rate of heat transfer for heat absorption case.

Fig. 5 deals with the variation of α only for the heat generation case while $Pr = 0.01$. When the amplitude of the wavy surface increases, near the leading edge, the rate of heat transfer $\theta'(x, 0)$ and its amplitude increase. But for increasing values of x, the rate of heat transfer and its amplitude reduce periodically.

Fig. 6 illustrates the effect of the heat generation/absorption parameter Q, on the development of streamlines which are plotted for $\alpha = 0.2$ and $Pr = 0.01$ where $\Delta\Psi = 2.5$. We observe that, in the case of heat generation $\Psi_{\max} = 67.6$ and for heat absorption case $\Psi_{\max} = 50.0$. This happens, because in the former case the buoyancy force increases, inducing the flow rate increases within the boundary layer. Consequently, the velocity distribution for the case of heat generation is higher than that of the heat absorption case.

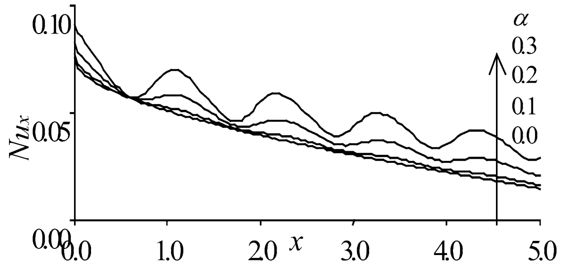

Fig. 5. Rate of heat transfer for different values of α while $Q > 0$ ($Q = 0.5$) and $Pr = 0.01$.

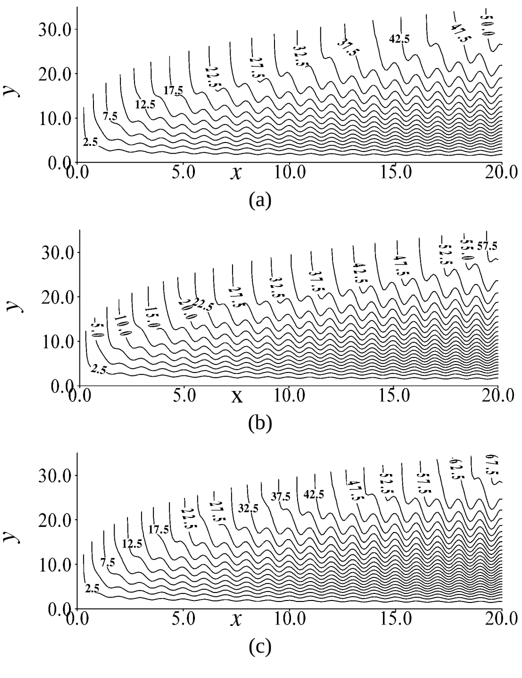

Fig. 6. Streamlines for (a) $Q < 0$ ($Q = -0.4$), (b) $Q = 0$ and (c) $Q > 0$ ($Q = 0.4$), respectively, while $\alpha = 0.2$ and $Pr = 0.01$.

The influence of the heat generation/absorption parameter Q on the isotherms profile for $\alpha = 0.2$ and $Pr = 0.01$ where $\Delta\theta = 0.06$ are shown in Fig. 7. As mentioned before, owing to the presence of the heat generation effect ($Q > 0$), the thermal state of the fluid increases, causing the thermal boundary layer to increase. In this case, in the down stream region the temperature variation is negligible. For heat absorption, we observe that the opposite phenomenon happens.

Figs. 8 and 9 depict the streamlines and isotherms for the values of α equal to 0.0, 0.1, 0.2 and 0.3, respectively, while $Q > 0$ and $Pr = 0.01$ where $\Delta\Psi = 2.5$ and $\Delta\theta = 0.06$. We observe that as the values of α increases, the maximum values of ψ decrease steadily. Finally we conclude that for much roughness of the surface, the velocity of fluid flow decreases in the boundary layer. Again from Fig. 9 we see that for wavy surface the isotherms take the wavy form and the amplitude of the isotherms increases if we keep on increasing the values of α. For increasing values of α, the thermal boundary layer thickness slightly decreases.

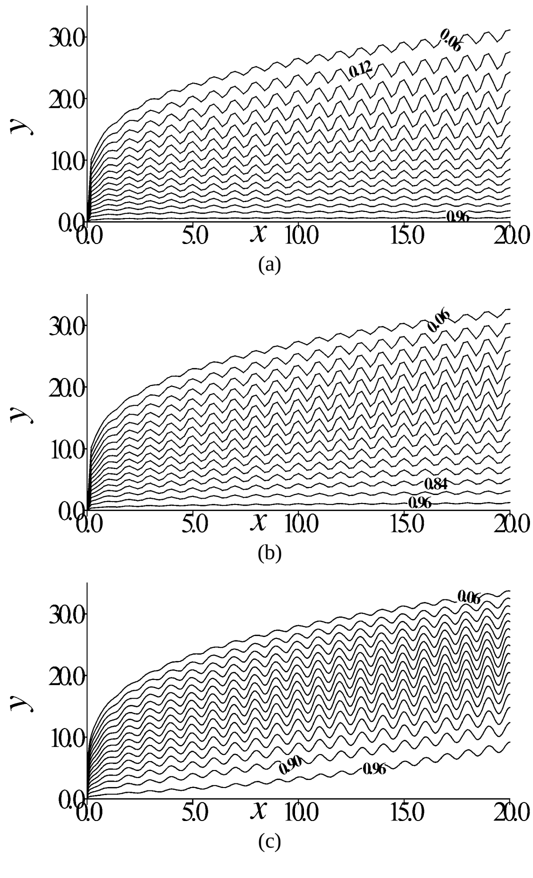

Fig. 7. Isotherms for (a) $Q < 0$ ($Q = -0.4$), (b) $Q = 0$ and (c) $Q > 0$ ($Q = 0.4$), respectively, while $\alpha = 0.2$ and $Pr = 0.01$.

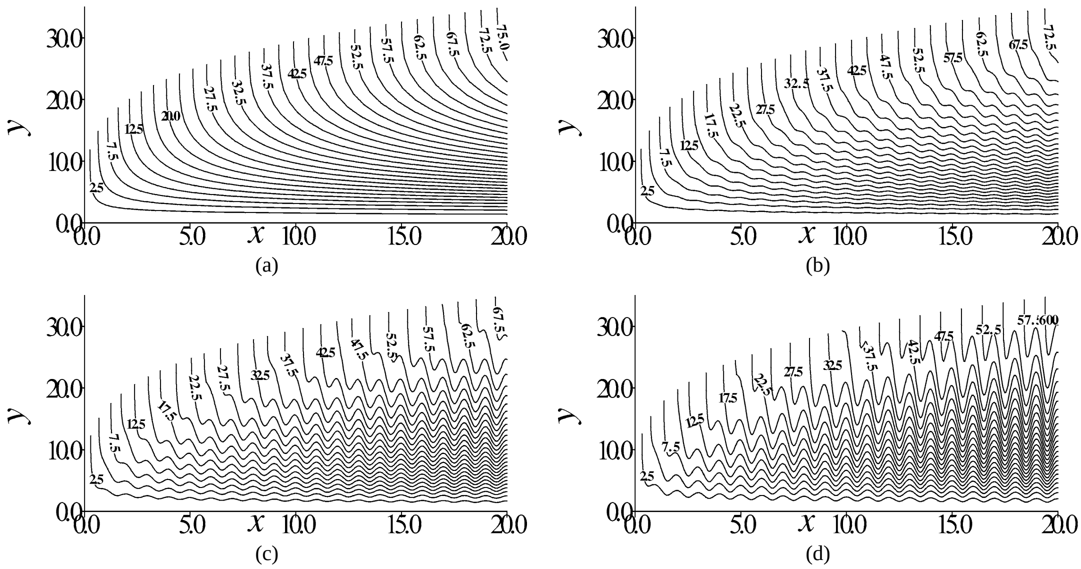

Fig. 8. Streamlines for (a) $\alpha = 0.0$, (b) $\alpha = 0.1$, (c) $\alpha = 0.2$, (d) $\alpha = 0.3$, while $Q > 0$ ($Q = 0.4$) and $Pr = 0.01$.

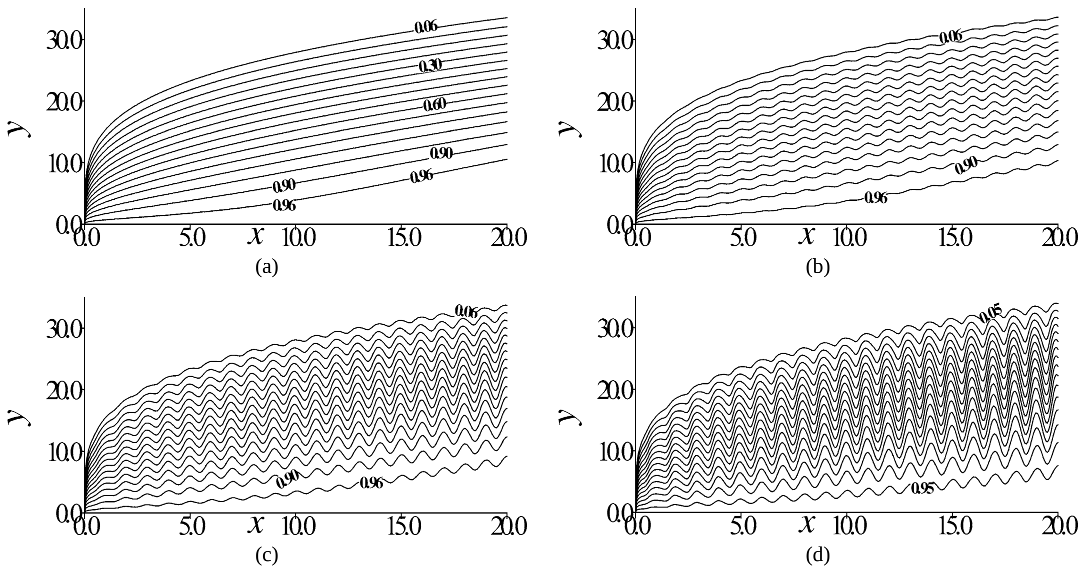

Fig. 9. Isotherms for (a) $\alpha = 0.0$, (b) $\alpha = 0.1$, (c) $\alpha = 0.2$, (d) $\alpha = 0.3$, while $Q > 0$ ($Q = 0.4$) and $Pr = 0.01$.

4. Conclusions

The effect of heat generation/absorption on natural convection boundary layer flow along a uniformly heated vertical wavy surface has been studied numerically. New variables to transform the complex geometry into a simple shape and were used a very efficient implicit finite-difference method known as Keller box scheme was employed to solve the boundary-layer equations. From the present investigation; we may conclude that, in the heat generation case, the thermal state of the fluid increases, consequently the Nusselt number, $Nu_x(Gr/x)^{-1/4}$ decreases when the axial distance variable x increases. For this case, the buoyancy force increases that increase the flow rate in the boundary layer. Also that leads to thickening the velocity boundary layer and thermal boundary layer. The opposite phenomenon occurs for the heat absorption case. The amplitude of the Nusselt number decreases in the downstream region for both the cases.

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